1^{st} Vienna Central European Seminar on Particle Physics and Quantum Field Theory

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BPS-Kink and more global solutions of the Chern-Simons (Super)Gravity Term

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paper: *L. Bergamin, DG, A. Iorio and C. Nuñez,* "Chemistry of the Chern-Simons Supergravity term: reduction to BPS kink, oxidation to M-theory and thermodynamical aspects," **hep-th/0409273**

1. The gravitational Chern-Simons term

2. Kaluza-Klein reduction

3. Supersymmetrization

4. All classical solutions

5. The BPS-kink

6. Hawking temperature

7. Oxidation to higher D

8. Open problems

The gravitational Chern-Simons (gCS) term

$$\mathcal{S}_{\mathrm{CS}} = \frac{1}{4\pi^2}\int \mathrm{d}^3x\epsilon^{\mu\nu\lambda}\left(\frac{1}{2}\Gamma^{\rho}_{\mu\sigma}\partial_\nu\Gamma^{\sigma}_{\lambda\rho} + \frac{1}{3}\Gamma^{\rho}_{\mu\sigma}\Gamma^{\sigma}_{\nu\tau}\Gamma^{\tau}_{\lambda\rho}\right)$$

where

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}G^{\lambda\rho}(\partial_\mu G_{\nu\rho} + \partial_\nu G_{\mu\rho} - \partial_\rho G_{\mu\nu})$$

and $G_{\mu\nu}$ is the 3-dimensional metric tensor

NOT the Witten action (EH as CS in 3D):

- Witten: like Palatini in 4D: depends on Dreibein *and* connection, on shell equivalent to EH, EOM: $R_{\mu\nu} = 0$
- gCS: depends on metric $G_{\mu\nu}$ only (via connection ${\Gamma^{\lambda}}_{\mu\nu}$), EOM: $C_{\mu\nu} = 0$
- Witten+gCS: DJT, “massive graviton”

S. Deser, R. Jackiw and S. Templeton, Ann. Phys. **281** *(2000) 409-449* (original article from 1982)
E. Witten, Nucl. Phys. **B311** *(1988) 46*

Kaluza-Klein reduction to 2D

$$G_{\mu\nu} = \begin{pmatrix} g_{mn} - \varphi a_m a_n & -\varphi a_m \\ -\varphi a_n & -\varphi \end{pmatrix}$$

- g_{mn} is the D=2 metric (r: curvature)
- a_m is a D=2 gauge vector (f: field strength)
- φ is a scalar (conformal factor)

In frame $\varphi = 1$ KK reduction yields

$$\mathcal{S}_{GIJP} = -\frac{1}{8\pi^2} \int \mathrm{d}^2 x \sqrt{-g}\, (fr + f^3)$$

Note: $\mathbb{Z}_2$ symmetry of EOM: $f \to -f$

G. Guralnik, A. Iorio, R. Jackiw and S.Y. Pi, `hep-th/0305117`

DG and W. Kummer, `hep-th/0306036` (first order)

Supersymmetrization via gPSM

Equivalence to graded Poisson-σ model

$$\mathcal{S}_{gPSM} = \int_{\mathcal{M}_2} \mathrm{d}X^I \wedge A_I + \frac{1}{2} P^{IJ} A_J \wedge A_I \,.$$

- gauge field 1-forms: $A_I = (\omega, e_a, \psi_\alpha, A)$, connection, Zweibeine, gravitini, $U(1)$

- target space coordinates: $X^I = (X, X^a, \chi^\alpha, Y)$, dilaton, Lmf. torsion, dilatini, $U(1)$ charge

- Poisson tensor: dimension (kernel) = 2, fixed uniquely by pre-potential $u(X, Y)$

Result for the pre-potential describing SUCS:

$$u(X, Y) = X^2 - Y$$

P. Schaller and T. Strobl, `hep-th/9405110` (bosonic precursor); *M. Ertl, W. Kummer and T. Strobl,* `hep-th/0012219`*;* *L. Bergamin and W. Kummer,* `hep-th/0209209`, `hep-th/0306217`*;* *DG, L. Bergamin and W. Kummer,* `hep-th/0310006`

SUSY trafos:

$$\delta e_m{}^a \quad \propto \quad \underbrace{\varepsilon\Gamma^a\psi_m}_{\text{standard}}$$

$$\delta a_m \quad \propto \quad \underbrace{\varepsilon\Gamma_2\psi_m}_{\text{remembers } D=3}$$

$$\delta\psi_{m\alpha} \quad \propto \quad \underbrace{\hat{D}_m\varepsilon_\alpha}_{\text{see below}}$$

where

$$\hat{D}_m\varepsilon_\alpha = \underbrace{\partial_m\varepsilon_\alpha + \frac{1}{2}\omega_m(\Gamma_2\varepsilon)_\alpha}_{\text{standard terms in } D=2} + \frac{i}{2}F(\Gamma_m\varepsilon)_\alpha$$

$$F = f + \epsilon^{mn}\psi_n\Gamma_2\psi_m$$

$\frac{1}{2}\Gamma_2$ is the 2-dimensional generator of Lorentz transformations in spinor space

$\Gamma_m = e_m^a\Gamma_a$ (with "flat" Γ_a)

"Expected" SUSY trafos from pre-potential: non-trivial statement

Pre-potential fixed by *bosonic* part already!

L. Bergamin, DG, A. Iorio, C. Nuñez, hep-th/0409273

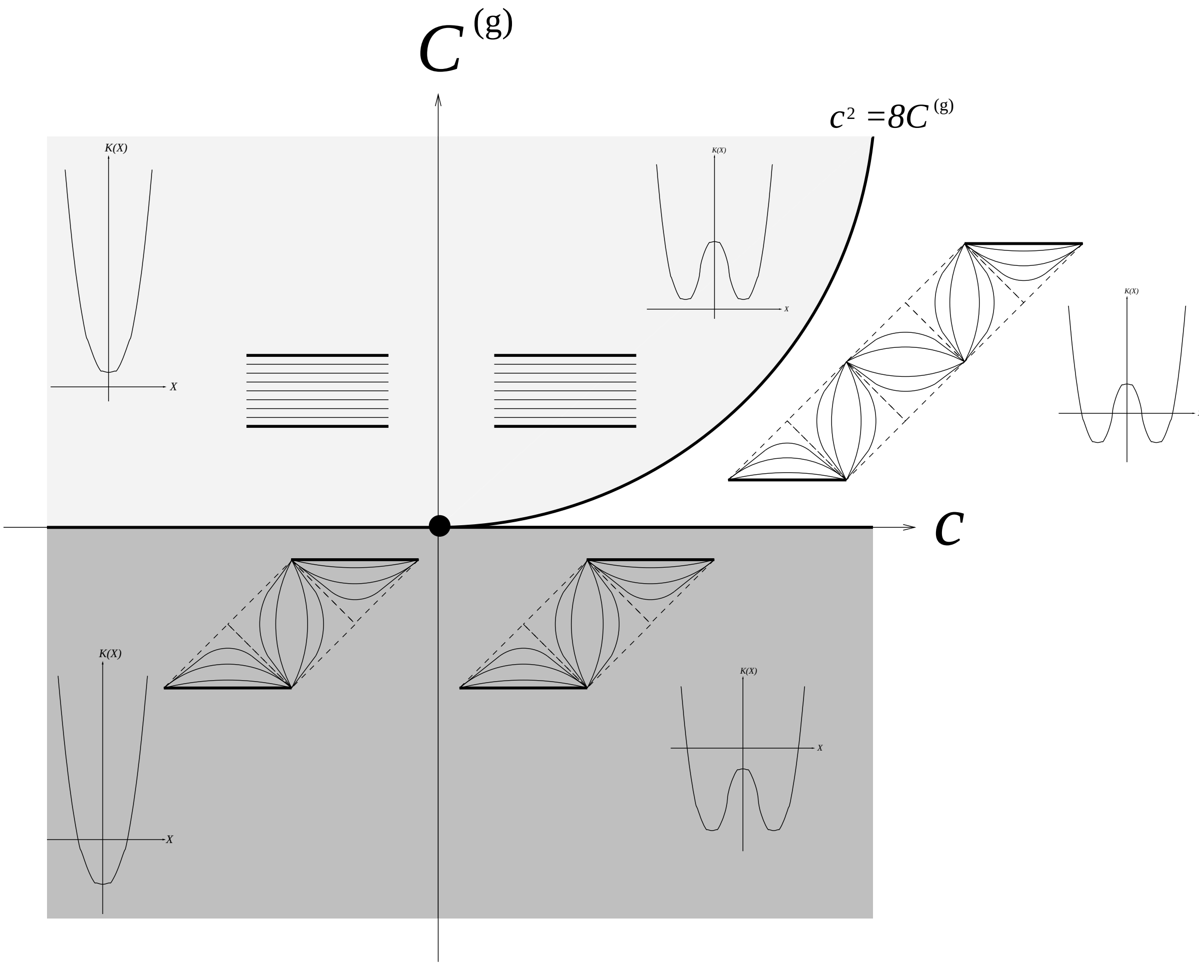
All classical solutions (bosonic)

Standard methods: line element in EF patch:

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\;\mathrm{d}X + \mathrm{Killing}(X; c, \mathcal{C}^{(g)})\,\mathrm{d}u^2$$

c: charge, $\mathcal{C}^{(g)}$: mass, Killing norm:

$$\mathrm{Killing}(X; c, \mathcal{C}^{(g)}) = \frac{1}{4}X^4 - \frac{1}{2}c\,X^2 + 2\,\mathcal{C}^{(g)}$$



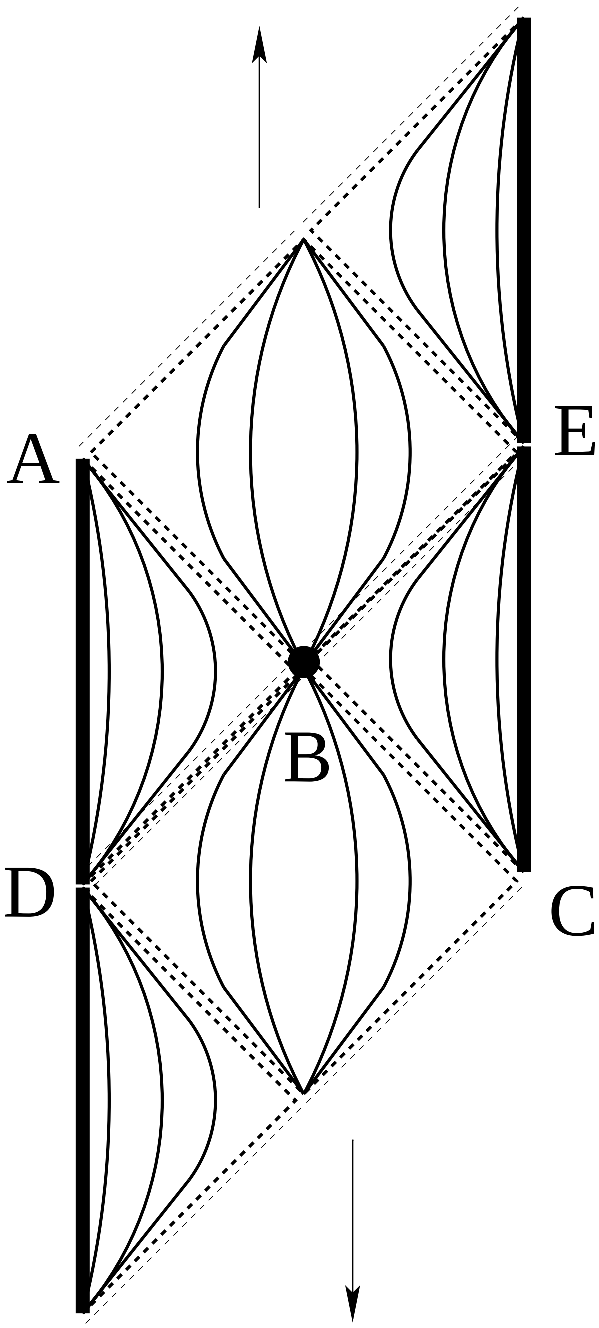
DG, W. Kummer, D. Vassilevich, hep-th/0204253, (2D dilaton gravity); *DG, W. Kummer,* hep-th/0306036

The BPS-kink

$$\text{Killing}_{BPS}(X;c) \propto (X^2 - c)^2 \geq 0$$

- half SUSY solution ($N = (1,1)$ SUGRA)
- extremal Killing horizons only
- ground state solution
- connects $\mathbb{Z}_2$ symmetry breaking AdS_2 vacua
- patching: induces self dual matter flux
- conformal trafo: asymptotically AdS_2 (without induced matter flux!)

G. Guralnik, A. Iorio, R. Jackiw and S.Y. Pi, hep-th/0305117
DG and W. Kummer, hep-th/0306036
L. Bergamin, DG, A. Iorio, C. Nuñez, hep-th/0409273



Fluxes: from A,C,D,E to B along extremal Killing horizons

Triangular patches: AdS_2 (no fluxes: as. AdS_2), Square patches: BPS-kink

Hawking temperature

From surface gravity ($M = 0$ for BPS):

$$T_H = \frac{1}{4\pi}\left|\frac{\mathrm{d}}{\mathrm{d}X}\text{Killing}(X; C, c)\right|_{X=X_{\text{horizon}}}$$
$$= \text{see graph on next page}$$

Specific heat:

$$C_s = \frac{\mathrm{d}M}{\mathrm{d}T_H} = \gamma(M) T_H$$

For low T_H (low M): Sommerfeld constant:

$$\gamma(M \to 0) = \frac{4\pi^2}{c}$$

For general M ("Sommerfeld function"):

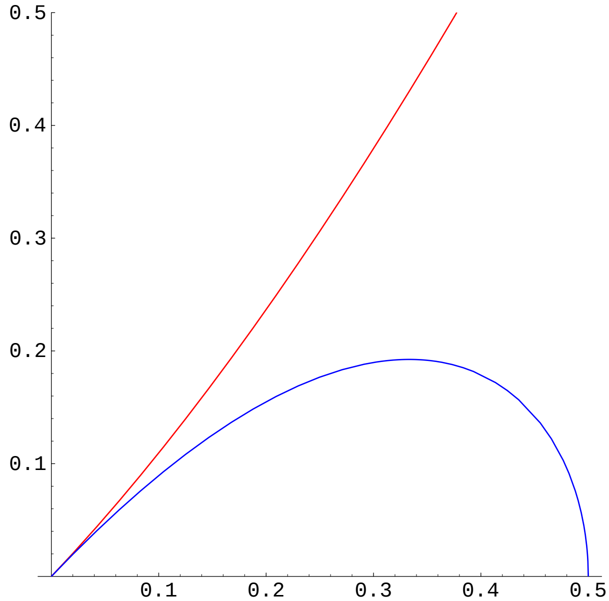
$$\gamma(M) = \text{see graph on next page}$$

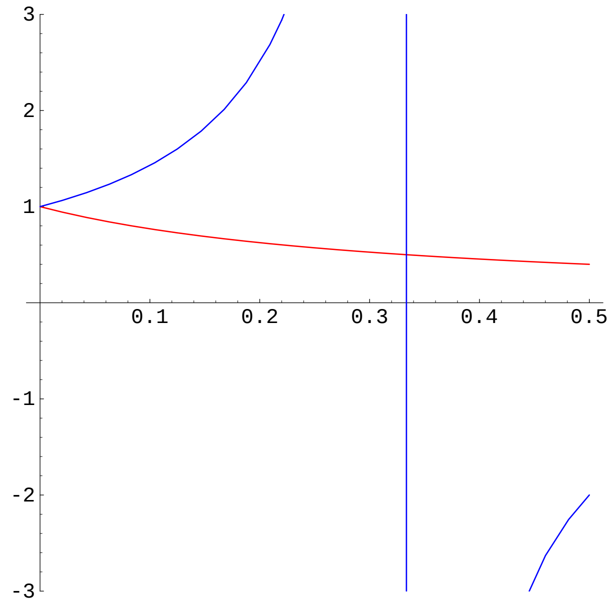
Hawking-Page like transition (inner horizon):

$$T_{\text{critical}} \propto c^{3/2}$$

S. Hawking and D. Page, Commun. Math. Phys. **87** *(1983) 577.*

Hawking temperature   Sommerfeld function





Expressed as functions of $\sqrt{2M}$

Red: outer horizon, Blue: inner horizon
Note: c has been set to 1 for the plots

Bekenstein-Hawking entropy for BPS:

$$S_{BH} \propto X_{\text{horizon}} \propto \sqrt{c}$$

J. Gegenberg, G. Kunstatter, D. Louis-Martinez, `gr-qc/9408015`; *W. Kummer, D. Vassilevich,* `gr-qc/9907041`
DG, W. Kummer, D. Vassilevich, `hep-th/0204253`
L. Bergamin, DG, A. Iorio, C. Nuñez, `hep-th/0409273`

Oxidation to D=3,4 and 11

$D = 3$ (*S. Deser, J. Kay, Phys. Lett.* **B120** *(1983) 97*)

- 2D BPS is not 3D BPS

- asymptotically AdS, but not $AAdS$ (NLO)

$D = 4$ (*S.L. Cacciatori et. al.*, `hep-th/0406238`)

- some SUSY solutions of 4D $U(1)$ gauged SUGRA: same EOM as KK reduced CS

- soliton of photons stabilized by gravity

$D = 11$ (via *M. Cvetic et. al.*, `hep-th/9903214`)

- lifting prescription: $AdS_4 \times S^7$

- metric may be $AAdS$: AdS/CFT aspects!

K. Skenderis, `hep-th/0209067` (and refs. therein)

Open problems

- in D=2: other derivation of T_H

- in D=2: coupling to matter

- in D=3: generalize to notion of $\underbrace{AAAdS}_{\text{almost } AAdS}$?

- in D=3: study different SUSY

- in D=4: understand equivalence to KK reduced CS – more general?

- in D=4: check $AAdS$ property

- in D=11: study AdS/CFT aspects

Summary of literature:

Chern-Simons gravity:
S. Deser, R. Jackiw and S. Templeton, Ann. Phys. **281** (2000) 409-449 (original article from 1982); E. Witten, Nucl. Phys. **B311** (1988) 46; P. van Nieuwenhuizen, Phys. Rev. **D32** (1985) 872; A. Achucarro and P.K. Townsend, Phys. Lett. **B180** (1986) 89; J.H. Horne, E. Witten, Phys. Rev. Lett. **62** (1989) 501.

Chern-Simons supergravity:
S. Deser, J. Kay, Phys. Lett. **B120** (1983) 97.

2D dilaton gravity:
DG, W. Kummer, D. Vassilevich, `hep-th/0204253`.

gPSM/SUGRA in first order formalism:
P. Schaller and T. Strobl, `hep-th/9405110` (bosonic precursor); M. Ertl, W. Kummer and T. Strobl, `hep-th/0012219`; L. Bergamin and W. Kummer, `hep-th/0209209`, `hep-th/0306217`; DG, L. Bergamin and W. Kummer, `hep-th/0310006`

Higher D, holography:
S.L. Cacciatori et. al., `hep-th/0406238` (D=4); M.M. Caldarelli, `hep-th/0411153` (D=4); M. Cvetic et. al., `hep-th/9903214` (D=11); K. Skenderis, `hep-th/0209067` (and refs. therein; holographic renormalization)

KK reduced CS (bosonic):
G. Guralnik, A. Iorio, R. Jackiw and S.Y. Pi, `hep-th/0305117`
DG and W. Kummer, `hep-th/0306036` (first order)

This talk: KK reduced CS (SUGRA):

L. Bergamin, DG, A. Iorio, C. Nuñez, `hep-th/0409273`